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# The Bianchi classification of maximal $D = 8$ gauged supergravities

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## Abstract

We perform the generalized dimensional reduction of  $D = 11$  supergravity over three-dimensional group manifolds as classified by Bianchi. Thus, we construct 11 different maximal  $D = 8$  gauged supergravities, two of which have an additional parameter. One class of group manifolds (class B) leads to supergravities that are defined by a set of equations of motion that cannot be integrated to an action. All 1/2 BPS domain wall solutions are given. We also find a non-supersymmetric domain wall solution where the single transverse direction is time. This solution describes an expanding universe and upon reduction gives the Einstein–de Sitter universe in  $D = 4$ . The uplifting of the different solutions to M-theory and the isometries of the corresponding group manifold are discussed.

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## 1. Introduction

The first example of a maximal  $D = 8$  gauged supergravity is the  $SO(3)$  gauged supergravity constructed in 1985 by Salam and Sezgin [1]. In recent years this maximal  $D = 8$  gauged supergravity has regained interest for several reasons. First of all the  $D = 8$  theory was used in the construction of the dyonic membrane [2]. It also occurs in the DW/QFT correspondence when one considers the near-horizon limit of the D6-brane [3]. Soon after, a number of papers appeared where maximal  $D = 8$  gauged supergravity played an important role in the construction of special holonomy manifolds by considering wrapped branes (see, e.g., [4–7]). More recently, the same theory also turned up in a discussion of gravitational topological quantum field theories [8] and accelerating universes [9].

In view of all these applications, it is of interest to ask oneself how unique  $SO(3)$  gauged supergravity is. In a recent paper [10], we performed a generalized dimensional reduction of  $D = 11$  supergravity over three-dimensional group manifolds [11, 12]<sup>4</sup>. By using one class of group manifolds, called class A in [14], we constructed five other maximal  $D = 8$  supergravities with gauge groups  $SO(2, 1)$ ,  $ISO(2)$ ,  $ISO(1, 1)$ ,  $Heis_3$  and  $U(1)^3$ . Here  $Heis_3$  denotes the three-dimensional Heisenberg group (with generators corresponding to position, momentum and identity). The theory with gauge group  $U(1)^3$  is obtained from a reduction over a torus  $T^3$  and is referred to as the ungauged theory, since there are no fields that carry any of the  $U(1)$  charges. All groups mentioned above are related to  $SO(3)$  by group contraction and/or analytic continuation. We will refer to them as class A supergravities.

In the same paper we showed that by using another class of group manifolds, called class B in [14], yet more gauged supergravities can be constructed whose gauge groups can be seen as extensions of  $ISO(2)$ ,  $ISO(1, 1)$ ,  $Heis_3$  and  $U(1)^3$ . We call these class B supergravities. There is extensive literature on the fact that a class B group manifold reduction leads to inconsistent field equations when reducing the action a fact first noticed by Hawking in [15] and discussed in [16] (recent overviews are given in [17, 18]). This is related to the fact that the field equations following from the reduced action do not coincide with the reduction of the field equations themselves [19]. We find, by explicitly performing the group manifold procedure, that the reduction can be performed on-shell, i.e. at the level of the equations of motion or the supersymmetry transformations. Particularly in string theory this seems to be a relevant approach since the worldsheet theory yields spacetime field equations rather than an action principle.

Making use of the fact that the class B group manifolds have two commuting isometries, we provide an alternative way of viewing this issue by relating the class B group manifold reduction to a Scherk–Schwarz reduction [20] from nine dimensions. In this procedure one uses an internal scale symmetry that leaves the  $D = 9$  equations of motion invariant but scales the Lagrangian. We indicate the M-theory origin of this scale symmetry. The fact that such a scale symmetry of the equations of motion can be used for a Scherk–Schwarz reduction and leads to equations of motion that cannot be integrated to a Lagrangian was first observed in [21].

Supergravities without an action naturally occur in the free differentiable algebra approach to supergravity (see, e.g., [22]). Furthermore, they have occurred recently in a study of matter-coupled supergravities in  $D = 5$  dimensions [23]. We note that, although the  $D = 8$  class B supergravities have no Lagrangian, there is a hidden Lagrangian in the sense that these theories can be obtained by dimensional reduction of a theory in  $D = 11$  dimensions with a Lagrangian.

Note that among the different group manifolds there is a number of non-compact manifolds, in particular all class B group manifolds. Thus, many of the reductions we perform are not compactifications in the usual sense (on a small internal manifold) but rather consistent truncations of the full higher-dimensional theory to a lower-dimensional subsector. The consistency of the truncation guarantees that  $D = 8$  solutions uplift to  $D = 11$  solutions. We do not consider global issues here and focus on local properties.

In [10], we showed that for the five maximal  $D = 8$  class A gauged supergravities, the most general domain wall solution is given by a so-called  $n$ -tuple domain wall solution with  $n \leq 3$ , which can be viewed as the superposition of  $n$  domain walls. The embedding of this  $n$ -tuple domain wall solution into M-theory for the  $SO(3)$  case naturally includes the near-horizon limit of the Kaluza–Klein monopole, as conjectured by the DW/QFT correspondence. In [3],

<sup>4</sup> The construction of gauged maximal supergravities (with a Lagrangian) in diverse dimensions is discussed from a purely group-theoretical point of view in [13].

the  $SO(3)$  domain wall with  $n = 1$  was found to correspond to the one-centre monopole, while we found in [10] that the  $SO(3)$  domain wall with  $n = 2$  uplifted to the near-horizon limit of the two-centre monopole solution. Both these solutions are non-singular upon uplifting whereas the  $n = 3$  domain wall uplifts to a singular spacetime [24].

In this paper we continue the work of [10]. In particular, we construct the supersymmetry rules of maximal  $D = 8$  gauged supergravities for both classes A and B by dimensionally reducing the  $D = 11$  supergravity over a three-dimensional group manifold [11, 12]. In this procedure the three-dimensional Lie algebra defining the group manifold becomes the algebra of the gauge group after reduction, and it is due to this approach that the classification of the eight-dimensional gauged supergravities coincides with the Bianchi classification of three-dimensional Lie algebras [25]. The reduction gives rise to 11 different maximal  $D = 8$  gauged supergravities, two of which have an additional parameter. We show how all these theories, except those whose gauge group is simple (i.e.  $SO(3)$  or  $SO(2, 1)$  for three-dimensional groups), can be obtained by a generalized reduction of maximal  $D = 9$  ungauged supergravity using its global symmetry group [20].

After constructing the different theories, we investigate the  $1/2$  BPS domain wall solutions for both classes A and B. We also discuss the isometries of the corresponding group manifold and find that the class A  $n$ -tuple domain wall solution of [10] gives a natural realization of isometry enhancement on a group manifold as discussed in Bianchi's paper [25]. We find that the domain wall solutions of two class A supergravities allow for the maximum number of isometries. In addition, we identify the remaining class A solution with six isometries.

The Bianchi classification suggests that there are two further solutions with maximum number of isometries for class B supergravities. We show that these indeed exist and are given by the same solution. This is a spacelike domain wall solution, i.e. a domain wall solution where the single transverse direction is time, and it describes an expanding universe with the same qualitative features as those of the Einstein–de Sitter universe. By instead reducing over a seven-dimensional group manifold, we find the Einstein–de Sitter universe in  $D = 4$ , which might be an acceptable model of our universe [26]. The uplifting of the different domain wall solutions to M-theory is discussed.

The outline of the paper is as follows. In section 2, we review the Bianchi classification of three-dimensional Lie groups and group manifolds. In section 3, we perform the dimensional reduction of  $D = 11$  supergravity over a three-dimensional group manifold, thereby constructing 11 different maximal  $D = 8$  gauged supergravities. In section 4, we show that each of these theories, except the  $SO(3)$  and  $SO(2, 1)$  cases, can be obtained by a generalized reduction of the unique maximal  $D = 9$  ungauged supergravity. Various solutions of these theories are discussed in section 5. Our conclusions are given in section 6. There are two appendices. Appendix A gives the supersymmetry rules of the  $D = 8$  theories we construct in this work, while appendix B gives the explicit expressions of the Killing vectors corresponding to the isometry enhancement of the group manifolds for the Bianchi types relevant for the solutions we consider.

## 2. Bianchi classification of 3D groups and manifolds

In this section, we review the Bianchi classification [25] of three-dimensional Lie groups and discuss how these can be realized as isometries on three-dimensional Euclidean manifolds<sup>5</sup>.

<sup>5</sup> The classification method used nowadays and presented here is not Bianchi's original one, but is due to Schücking and Behr (see Kundt's paper based on the notes taken in a seminar given by Schücking [27], and the editorial notes [28]), and the earliest publications in which this method is followed are [14, 29]. The history of the classification of three- and four-dimensional real Lie algebras is also reviewed in [30].

We assume that the generators of the three-dimensional Lie group satisfy the commutation relations ( $m = 1, 2, 3$ )

$$[T_m, T_n] = f_{mn}{}^p T_p, \quad (2.1)$$

with constant structure coefficients  $f_{mn}{}^p$  subject to the Jacobi identity  $f_{[mn}{}^q f_{p]q}{}^r = 0$ . For three-dimensional Lie groups, the structure constants have nine components, which can be conveniently parametrized by

$$f_{mn}{}^p = \epsilon_{mnq} Q^{pq} + 2\delta_{[m}{}^p a_{n]}, \quad Q^{pq} a_q = 0. \quad (2.2)$$

Here  $Q^{pq}$  is a symmetric matrix with six components, and  $a_m$  is a vector with three components. The constraint on their product follows from the Jacobi identity. Having  $a_q = 0$  corresponds to an algebra with traceless structure constants:  $f_{mn}{}^n = 0$ . Following [14] we distinguish between class A and class B algebras which have vanishing and non-vanishing traces, respectively.

Of course Lie algebras are only defined up to changes of basis,  $T_m \rightarrow R_m{}^n T_n$ , with  $R_m{}^n \in GL(3, \mathbb{R})$ . The corresponding transformation of the structure constants and their components reads

$$f_{mn}{}^p \rightarrow f'_{mn}{}^p = R_m{}^q R_n{}^r (R^{-1})_s{}^p f_{qr}{}^s : \quad \begin{cases} Q^{mn} \rightarrow \det(R) ((R^{-1})^T Q R^{-1})^{mn}, \\ a_m \rightarrow R_m{}^n a_n. \end{cases} \quad (2.3)$$

These transformations are naturally divided into two complementary sets. First there is the group of automorphism transformations with  $f_{mn}{}^p = f'_{mn}{}^p$ , whose dimension is given in table 1 for the different algebras and which are described in [31]. Then there are the transformations that change the structure constants, and these can always be used [32, 33] to transform  $Q^{pq}$  into a diagonal form and  $a_q$  to have only one component. We will explicitly go through the argument.

Consider an arbitrary symmetric matrix  $Q^{mn}$  with eigenvalues  $\lambda_m$  and orthogonal eigenvectors  $\vec{u}_m$ . Taking

$$R^T = (\sqrt{d_2 d_3} \vec{u}_1, \sqrt{d_1 d_3} \vec{u}_2, \sqrt{d_1 d_2} \vec{u}_3), \quad (2.4)$$

with  $d_m \neq 0$  and  $\text{sgn}(d_1) = \text{sgn}(d_2) = \text{sgn}(d_3)$  we find that

$$Q^{mn} \rightarrow \text{diag}(d_1 \lambda_1, d_2 \lambda_2, d_3 \lambda_3). \quad (2.5)$$

We now distinguish between four cases, depending on the rank of  $Q^{mn}$ .

- Rank ( $Q^{mn}$ ) = 3. In this case all components of  $a_m$  necessarily vanish (due to the Jacobi identity), and we can take  $d_m = \pm 1/|\lambda_m|$  to obtain

$$Q^{mn} = \pm \text{diag}(\text{sgn}(\lambda_1), \text{sgn}(\lambda_2), \text{sgn}(\lambda_3)), \quad a_m = (0, 0, 0). \quad (2.6)$$

- Rank ( $Q^{mn}$ ) = 2. In this case one eigenvalue vanishes which we take to be  $\lambda_1$ . Then we set  $d_i = \pm 1/|\lambda_i|$ , with  $i = 2, 3$ , to obtain  $Q^{mn} = \pm \text{diag}(0, \text{sgn}(\lambda_2), \text{sgn}(\lambda_3))$ . From the Jacobi identity, it then follows that  $a_m = (a, 0, 0)$ . We distinguish between vanishing and non-vanishing vectors. In the case  $a \neq 0$ , one might think that one can use  $d_1$  to set  $a = 1$ , but from the transformation rule of  $a_m$  (2.3) and the form of  $R$  (2.4) it can be seen that  $a \sim \sqrt{d_2 d_3}$ , and therefore  $a$  cannot be fixed by  $d_1$ . In this case we thus have a one-parameter family of Lie algebras:

$$Q^{mn} = \pm \text{diag}(0, \text{sgn}(\lambda_2), \text{sgn}(\lambda_3)), \quad \begin{cases} a_m = (0, 0, 0), \\ a_m = (a, 0, 0). \end{cases} \quad (2.7)$$

**Table 1.** The Bianchi classification of three-dimensional Lie algebras in terms of the components of their structure constants. Note that there are two one-parameter families  $\text{VI}_a$  and  $\text{VII}_a$  with special cases  $\text{VI}_0$ ,  $\text{VII}_0$  and  $\text{VI}_{a=1/2} = \text{III}$ . The algebra  $\text{heis}_3$  denotes the three-dimensional Heisenberg algebra. The table also gives the dimensions of the automorphism groups and the dimensions of the possible isometry groups of the corresponding group manifolds. The identifications in column 5 can be found in [34].

Bianchi	$a$	$(q_1, q_2, q_3)$	Class	Algebra	Dim(Aut)	Dim(Iso)
I	0	(0, 0, 0)	A	$u(1)^3$	9	6
II	0	(0, 0, 1)	A	$\text{heis}_3$	6	4
III	$\frac{1}{2}$	(0, -1, 1)	B		4	4
IV	1	(0, 0, 1)	B		4	3
V	1	(0, 0, 0)	B		6	6
$\text{VI}_0$	0	(0, -1, 1)	A	$\text{iso}(1, 1)$	4	3
$\text{VI}_a$	$a$	(0, -1, 1)	B		4	3
$\text{VII}_0$	0	(0, 1, 1)	A	$\text{iso}(2)$	4	3, 6
$\text{VII}_a$	$a$	(0, 1, 1)	B		4	3, 6
VIII	0	(1, -1, 1)	A	$\text{so}(2, 1)$	3	3, 4
IX	0	(1, 1, 1)	A	$\text{so}(3)$	3	3, 4, 6

- Rank  $(Q^{mn}) = 1$ . In this case two eigenvalues vanish, e.g.  $\lambda_1 = \lambda_2 = 0$ . We set  $d_3 = \pm 1/|\lambda_3|$  to obtain  $Q^{mn} = \pm \text{diag}(0, 0, \text{sgn}(\lambda_3))$ . Again one distinguishes between  $a_m = 0$  and  $a_m \neq 0$ . In the latter case one is left with a vector  $a_m = (a_1, a_2, 0)$ , of which  $a_1 \sim \sqrt{d_2 d_3}$  and  $a_2 \sim \sqrt{d_1 d_3}$ . Thus, one can use  $d_1$  and  $d_2$  to adjust the length of  $\vec{a}$  to 1, after which an  $O(3)$  transformation in the  $(1, 2)$ -subspace gives the final result:

$$Q^{mn} = \pm \text{diag}(0, 0, \text{sgn}(\lambda_3)), \quad \begin{cases} a_m = (0, 0, 0), \\ a_m = (1, 0, 0). \end{cases} \quad (2.8)$$

- Rank  $(Q^{mn}) = 0$ . In this case all three eigenvalues vanish and therefore  $Q^{mn} = 0$ . Thus, the transformation with matrix (2.4) is irrelevant. For  $a_m \neq 0$ , it follows from (2.3) that one can first do a scaling to get  $|\vec{a}| = 1$  and then an  $O(3)$  transformation to obtain

$$Q^{mn} = \text{diag}(0, 0, 0), \quad \begin{cases} a_m = (0, 0, 0), \\ a_m = (1, 0, 0). \end{cases} \quad (2.9)$$

Thus, we find that the most general three-dimensional Lie algebras can be described by  $Q^{mn} = \frac{1}{2} \text{diag}(q_1, q_2, q_3)$  and  $a_m = (a, 0, 0)$ . In this basis the commutation relations take the form

$$[T_1, T_2] = \frac{1}{2} q_3 T_3 - a T_2, \quad [T_2, T_3] = \frac{1}{2} q_1 T_1, \quad [T_3, T_1] = \frac{1}{2} q_2 T_2 + a T_3. \quad (2.10)$$

The different three-dimensional Lie algebras are obtained by taking different signatures of  $Q^{mn}$  and are given in table 1. Naively one might conclude that the classification as given above leads to ten different algebras. However, it turns out that one has to treat the subcase  $a = 1/2$  of (2.7) as a separate case. We will come back to this case below when we discuss the isometries of the group manifold. Thus, the total number of inequivalent three-dimensional Lie algebras is 11, two of which are one-parameter families.

Of the 11 Lie algebras, only  $SO(3)$  and  $SO(2, 1)$  are simple while the rest are all non-semisimple [32, 35]. In the non-semisimple cases we always have  $q_1 = 0$ , for which choice the Abelian invariant subgroup is  $\{T_2, T_3\}$ , since  $T_1$  does not appear on the right-hand side in (2.10). All algebras of class A with traceless structure constants fall in the  $CSO(p, q, r)$ -classification

with  $p + q + r = 3$  as discussed in [10] and can give rise to compact and non-compact groups, while all algebras of class B correspond to non-compact groups [32].

In addition to the different three-dimensional Lie groups, one can consider their realizations as (a subgroup of) the isometry groups of three-dimensional Euclidean manifolds. It is well established [25] that, given an  $n$ -dimensional simply transitive group (which all the groups corresponding to type I up to type IX are), there is a corresponding  $n$ -dimensional manifold that allows this group as isometries. This manifold is called the group manifold. The manifold has by definition at least  $n$  isometries whose right-invariant Killing vectors  $X_a = X_a^b \partial / \partial z_b$  with  $a, b = 1, \dots, n$  satisfy

$$[X_a, X_b] = -f_{ab}^c X_c. \quad (2.11)$$

The full group of isometries may very well be bigger.

Let us first consider the case  $n = 2$ , i.e. two-dimensional manifolds. The isometry groups of surfaces are zero, one or three dimensional [25]. Thus, if one requires a two-dimensional simply transitive group to be realized as isometries on a two-dimensional manifold, one finds isometry enhancement: the full isometry group is necessarily three dimensional. Thus every two-dimensional group manifold has the maximum number of isometries and therefore constant curvature.

Turning to the case  $n = 3$ , the dimension of the isometry group,  $\text{Dim}(\text{Iso})$ , is restricted to 0, 1, 2, 3, 4 or 6 [14, 25, 36, 37]. Since we only consider group manifolds, on which a three-dimensional simply transitive group is realized as isometries, we have  $\text{Dim}(\text{Iso}) \geq 3$ . The three right-invariant Killing vectors (2.11) corresponding to these isometries are given in (3.14) and (3.15) (for our parametrization of the structure constants (2.10)). However, the full isometry group of the manifold may well be bigger. Consider as an example of such an isometry enhancement the group of Bianchi type I, i.e. the translation group in three dimensions. Its generators are the translational isometries of a flat manifold. The full isometry group of such a manifold is the six-dimensional  $ISO(3)$  group. Thus the isometry group of the Bianchi type I group manifold is always six dimensional.

In table 1 we give the dimension of the possible full isometry group of the group manifolds for all Bianchi types. For the simple Lie groups, i.e. those of type VIII and type IX, and for the non-semisimple Lie groups of type  $VII_0$  and type  $VII_a$ , there are different possibilities depending on the choice of the three-dimensional manifold, i.e. one can have isometry enhancement. Note that the group manifolds of types I, V,  $VII_0$ ,  $VII_a$  and IX allow for the maximum number of six isometries, in which case one is dealing with a manifold of constant curvature. For the one-parameter family of Lie algebras of type  $VI_a$  one has isometry enhancement for the value  $a = 1/2$ , which is the reason why it is treated as a separate case, i.e. type III = type  $VI_{a=1/2}$ . We will come back to the number of isometries of the group manifolds when discussing explicit solutions in section 5.

### 3. Reduction over a 3D group manifold

In this section we review the reduction of  $D = 11$  supergravity over a three-dimensional group manifold, leading to gauged supergravities in eight dimensions. We will follow [10] with emphasis on the new features when dealing with the Bianchi class B groups. To be precise, we get corrections proportional to the parameter  $a$  (the trace of the structure constants) to the supersymmetry transformation rules, which will be important when searching for solutions. This is discussed in more detail in section 5.



The reduction ansatz is formally the same for classes A and B and it involves the following fields:

$$\begin{aligned} 11D : \quad & \{\hat{e}_{\hat{\mu}}^{\hat{a}}, \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{\psi}_{\hat{\mu}}\}, \\ 8D : \quad & \{e_{\mu}^a, L_m^i, \varphi, \ell, A^m_{\mu}, V_{\mu mn}, B_{\mu\nu m}, C_{\mu\nu\rho}, \psi_{\mu}, \lambda_i\}, \end{aligned} \quad (3.1)$$

where the indices are defined according to an  $8 + 3$  split of the 11-dimensional spacetime:  $x^{\hat{\mu}} = (x^{\mu}, z^m)$  with  $\mu = (0, 1, \dots, 7)$  and  $m = (1, 2, 3)$ . Spacetime indices are written as  $\hat{\mu} = (\mu, m)$  while the tangent indices are  $\hat{a} = (a, i)$ . The three-dimensional space is taken to be a group manifold and we reduce over its three (non-Abelian) isometries.

Using a particular Lorentz frame, the reduction ansatz for the 11-dimensional bosonic fields is

$$\hat{e}_{\hat{\mu}}^{\hat{a}} = \begin{pmatrix} e^{-\frac{1}{6}\varphi} e_{\mu}^a & e^{\frac{1}{3}\varphi} L_m^i A^m_{\mu} \\ 0 & e^{\frac{1}{3}\varphi} L_n^i U^n_m \end{pmatrix}, \quad (3.2)$$

and

$$\hat{C}_{abc} = e^{\frac{1}{2}\varphi} C_{abc}, \quad \hat{C}_{abi} = L_i^m B_{abm}, \quad \hat{C}_{aij} = e^{-\frac{1}{2}\varphi} L_i^m L_j^n V_{amn}, \quad \hat{C}_{ijk} = e^{-\varphi} \epsilon_{ijk} \ell. \quad (3.3)$$

The reduction ansatz for the fermions, including the supersymmetry parameter  $\hat{\epsilon}$ , reads as follows:

$$\hat{\psi}_{\hat{a}} = e^{\varphi/12} (\psi_a - \frac{1}{6} \Gamma_a \Gamma^i \lambda_i), \quad \hat{\psi}_i = e^{\varphi/12} \lambda_i, \quad \hat{\epsilon} = e^{-\varphi/12} \epsilon. \quad (3.4)$$

The matrix  $L_m^i$  describes the five-dimensional  $SL(3, \mathbb{R})/SO(3)$  scalar coset space. It transforms under a global  $SL(3, \mathbb{R})$  acting from the left and a local  $SO(3)$  symmetry acting from the right. We take the following explicit representative, thus gauge fixing the local  $SO(3)$  symmetry:

$$L_m^i = \begin{pmatrix} e^{-\sigma/\sqrt{3}} & e^{-\phi/2+\sigma/2\sqrt{3}} \chi_1 & e^{\phi/2+\sigma/2\sqrt{3}} \chi_2 \\ 0 & e^{-\phi/2+\sigma/2\sqrt{3}} & e^{\phi/2+\sigma/2\sqrt{3}} \chi_3 \\ 0 & 0 & e^{\phi/2+\sigma/2\sqrt{3}} \end{pmatrix}, \quad (3.5)$$

which contains two dilatons,  $\phi$  and  $\sigma$ , and three axions  $\chi_m$ . It is convenient to define the local  $SO(3)$ -invariant scalar matrix

$$\mathcal{M}_{mn} = -L_m^i L_n^j \eta_{ij}, \quad (3.6)$$

where  $\eta_{ij} = -\mathbb{I}_3$  is the internal flat metric.

The only internal coordinate dependence in the ansatz appears via the matrix  $U^m_n$ , which is defined in terms of the left-invariant Maurer–Cartan 1-forms of a three-dimensional Lie group

$$\sigma^m \equiv U^m_n dz^n. \quad (3.7)$$

By definition these 1-forms satisfy the Maurer–Cartan equations

$$d\sigma^m = -\frac{1}{2} f_{np}^m \sigma^n \wedge \sigma^p, \quad f_{mn}^p = -2(U^{-1})^r_m (U^{-1})^s_n \partial_{[r} U^p_{s]}, \quad (3.8)$$

where the  $f_{mn}^p$  are independent of  $z^m$  and form the structure constants of the group manifold. Note that we use a slight extension of the original procedure of Scherk and Schwarz [12] by allowing for structure constants with non-vanishing trace (leading to class B supergravities). This corresponds to a group manifold which does not have a constant volume element. We find, by explicitly performing the group manifold procedure, that the class B reduction can be performed on-shell, i.e. at the level of the equations of motion or the supersymmetry variations, but not at the level of the action. Indeed, the lower-dimensional field equations cannot be integrated to an action.



An explicit representation of the Maurer–Cartan 1-forms for general rank of the matrix  $Q$  was given in [10] in the case of class A. Including class B, i.e.  $a \neq 0$ , leads to the following matrix:

$$U^m_n = \begin{pmatrix} 1 & 0 & -s_{1,3,2} \\ 0 & e^{az^1} c_{2,3,1} & e^{az^1} c_{1,3,2} s_{2,3,1} \\ 0 & -e^{az^1} s_{3,2,1} & e^{az^1} c_{1,3,2} c_{2,3,1} \end{pmatrix}, \quad (3.9)$$

where we have used the following abbreviations:

$$c_{m,n,p} \equiv \cos\left(\frac{1}{2}\sqrt{q_m}\sqrt{q_n}z^p\right), \quad s_{m,n,p} \equiv \sqrt{q_m} \sin\left(\frac{1}{2}\sqrt{q_m}\sqrt{q_n}z^p\right)/\sqrt{q_n}, \quad (3.10)$$

and it is understood that the structure constants satisfy the Jacobi identity, amounting to  $q_1 a = 0$ .

The relation between the Maurer–Cartan 1-forms  $\sigma^m$  and the three-dimensional isometry groups is as follows. The metric on the group manifold reads

$$ds^2 = e^{2\varphi/3} \mathcal{M}_{mn} \sigma^m \sigma^n, \quad (3.11)$$

where the scalars  $\varphi$  and  $\mathcal{M}$  are constants from the three-dimensional point of view. A vector field  $X$  defines an isometry if it leaves the metric invariant

$$\mathcal{L}_X g_{mn} = 0. \quad (3.12)$$

For all values of the scalars, the group manifold has three isometries generated by the right-invariant Killing vector fields. These fulfil the stronger requirement

$$\mathcal{L}_{X_m} \sigma^n = 0, \quad (3.13)$$

for all three Maurer–Cartan forms on the group manifold and generate the algebra as given in (2.11). In the class A case, i.e.  $a = 0$ , the right-invariant Killing vectors generating the three isometries are given by

$$\begin{aligned} X_1 &= \frac{c_{1,2,3}}{c_{1,3,2}} \frac{\partial}{\partial z^1} - s_{2,1,3} \frac{\partial}{\partial z^2} + \frac{c_{1,2,3} s_{3,1,2}}{c_{1,3,2}} \frac{\partial}{\partial z^3}, \\ X_2 &= \frac{s_{1,2,3}}{c_{1,3,2}} \frac{\partial}{\partial z^1} + c_{1,2,3} \frac{\partial}{\partial z^2} - \frac{s_{1,2,3} s_{1,3,2}}{c_{1,3,2}} \frac{\partial}{\partial z^3}, \\ X_3 &= \frac{\partial}{\partial z^3}, \end{aligned} \quad (3.14)$$

whereas in the class B case, i.e.  $q_1 = 0$  and  $a \neq 0$ , they are given by

$$\begin{aligned} X_1 &= \frac{\partial}{\partial z^1} - \left(az^2 + \frac{1}{2}q_2 z^3\right) \frac{\partial}{\partial z^2} + \left(\frac{1}{2}q_3 z^2 - az^3\right) \frac{\partial}{\partial z^3}, \\ X_2 &= \frac{\partial}{\partial z^2}, \quad X_3 = \frac{\partial}{\partial z^3}. \end{aligned} \quad (3.15)$$

Here,  $\partial/\partial z^2$  and  $\partial/\partial z^3$  are manifest isometries. This follows from the fact that the matrix  $U^n_m$  is independent of  $z^2$  and  $z^3$ .

With the above ansatz, class B gauged supergravities can be obtained. For our present purposes, it is enough to reduce the supersymmetry transformation rules. Since we are primarily interested in domain wall solutions, we will truncate the reduction to just include the following fields:  $g_{\mu\nu}$ ,  $L_m^i$  and  $\varphi$ . The resulting  $D = 8$  fermionic transformations are

$$\begin{aligned} \delta\psi_\mu &= 2\partial_\mu \epsilon - \frac{1}{2}\phi_\mu \epsilon + \frac{1}{2}\mathcal{Q}_\mu \epsilon + \frac{1}{24}e^{-\varphi/2} f_{ijk} \Gamma^{ijk} \Gamma_\mu \epsilon - \frac{1}{6}e^{-\varphi/2} f_{ij}{}^j \Gamma_\mu \Gamma^i \epsilon, \\ \delta\lambda_i &= -\mathcal{P}_{ij} \Gamma^j \epsilon - \frac{1}{3}\partial\varphi \Gamma_i \epsilon - \frac{1}{4}e^{-\varphi/2} (2f_{ijk} - f_{jki}) \Gamma^{jk} \epsilon. \end{aligned} \quad (3.16)$$

Note that there is only one term with an explicit dependence on the trace of the structure

constants, namely the last term in  $\delta\psi_\mu$ . The full supersymmetry rules, without truncation, can be found in appendix A.

The global duality group  $GL(3, \mathbb{R})$  acts on the indices  $m, n, p$  in the obvious way and its action is explicitly given in [10]. In the gauged theory this is in general no longer a symmetry since it does not preserve the structure constants. The unbroken part is exactly given by the automorphism group of the structure constants as given in table 1. Of course it always includes the gauge group, which is embedded in  $GL(3, \mathbb{R})$  via

$$g_n^m = e^{\lambda^k f_{kn}^m}, \quad (3.17)$$

where  $\lambda^k$  are the local parameters of the gauge transformations. However, the full automorphism group can be bigger; for instance, it is nine dimensional in the  $U(1)^3$  case. Of course this amounts to the fact that the ungauged  $D = 8$  theory has a  $GL(3, \mathbb{R})$  symmetry. Note that all other cases have  $\text{Dim}(\text{Aut}) < 9$  and thus break the  $GL(3, \mathbb{R})$  symmetry to some extent. For instance, the  $SO(1, 1)$  subgroup corresponding to the determinant of the  $GL(3, \mathbb{R})$  element is broken by all non-vanishing structure constants.

The  $GL(3, \mathbb{R})$  transformations are not the only symmetries of the ungauged theory. There are two more generators leading to the full  $U$ -duality group

$$SL(3, \mathbb{R}) \times SL(2, \mathbb{R}). \quad (3.18)$$

The  $SL(3, \mathbb{R})$  and the  $SO(1, 1)$  subgroup of  $SL(2, \mathbb{R})$  conspire to form the  $GL(3, \mathbb{R})$ . Its fate after a non-trivial gauging has been discussed above, giving rise to the automorphism groups. To understand the fate of the other subgroups of  $SL(2, \mathbb{R})$ , one needs to define the doublet

$$\vec{f}_{mn}{}^p = \begin{pmatrix} f_{mn}{}^p \\ 0 \end{pmatrix}. \quad (3.19)$$

Under a global  $SL(2, \mathbb{R})$  transformation the full theory is invariant up to a transformation of the structure constants:

$$\vec{f}_{mn}{}^p \rightarrow \Omega \vec{f}_{mn}{}^p, \quad \Omega \in SL(2, \mathbb{R}). \quad (3.20)$$

From this transformation, one can see that the  $SO(2)$  and  $SO(1, 1)$  subgroups of  $SL(2, \mathbb{R})$  are broken by any non-zero structure constants<sup>6</sup> and thus in all theories except the Bianchi type I. In contrast, the doublet of structure constants (3.19) is invariant under an  $\mathbb{R}$  subgroup of the  $SL(2, \mathbb{R})$  symmetry.

#### 4. Nine-dimensional origin

In this section, we will show that all  $D = 8$  gauged supergravities except those whose gauge group is simple, i.e.  $SO(3)$  or  $SO(2, 1)$ , can be obtained by a generalized reduction of maximal  $D = 9$  ungauged supergravity using its global symmetry group<sup>7</sup> [20]. This is possible since all these theories follow from the reduction over a non-semisimple group manifold with two commuting isometries. If these two isometries are manifest, as in (3.9) with  $q_1 = 0$ , one can first perform a Kaluza–Klein reduction over  $T^2$  to nine dimensions.

Restricting ourselves to only those symmetries that are not broken by  $\alpha'$ -corrections, the  $D = 9$  global symmetry group is given by

$$SL(2, \mathbb{R}) \times SO(1, 1). \quad (4.1)$$

<sup>6</sup> The type II, VI<sub>0</sub> and VII<sub>0</sub> theories are related via an  $SO(2)$  transformation of  $90^\circ$  to the Kaluza–Klein reduction of the  $D = 9$  gauged theories of [38, 39]. Moreover, after any  $SO(2)$  transformation the type II theory can only be further uplifted to the  $D = 10$  massive IIA theory (see, e.g., [40].)

<sup>7</sup> This is a different reduction ansatz from the group manifold procedure as discussed in section 3. It is based on internal rather than spacetime symmetries, see also [10] for a discussion.

**Table 2.** The different non-semisimple Bianchi types of  $D = 8$  gauged supergravities, resulting from reduction of  $D = 9$  ungauged supergravity by using different combinations of subgroups of the global symmetry groups in  $D = 9$ . Here  $\Omega$  and  $\Lambda$  denote the elements of  $SL(2, \mathbb{R})$  and  $SO(1, 1)$ , respectively.

$D = 9 \Rightarrow D = 8$ reduction ansatz	$\Lambda = 1$ ( $\Rightarrow$ class A)	$\Lambda \neq 1$ ( $\Rightarrow$ class B)
$\Omega = \mathbb{I}_2$	$I = U(1)^3$	V
$\Omega \in \mathbb{R}$	$II = \text{Heis}_3$	VI
$\Omega \in SO(1, 1)$	$VI_0 = ISO(1, 1)$	$III = VI_{a=1/2}, VI_a$
$\Omega \in SO(2)$	$VII_0 = ISO(2)$	$VII_a$

Here the duality group  $SL(2, \mathbb{R})$  is a symmetry of the action and is not violated by  $\alpha'$ -corrections, since it descends from the duality group  $SL(2, \mathbb{R})$  of type IIB string theory. We denote its elements by  $\Omega$ . The explicit  $SO(1, 1)$  with elements  $\Lambda$  is a symmetry of the equations of motion only. Since it has an M-theory origin as the scaling symmetry<sup>8</sup>  $x^{\hat{\mu}} \rightarrow \Lambda x^{\hat{\mu}}$  for  $\hat{\mu} = 10, 11$ , this is not violated by  $\alpha'$ -corrections either. This  $SO(1, 1)$  is precisely the scale transformation with parameter  $\Lambda = \exp(a\tau^1)$ , generated by the matrix  $U^m_n$ , see (3.9), for  $q_1 = q_2 = q_3 = 0$ . Note that this scaling symmetry scales the volume element of the torus, which explains why it is only a symmetry of the  $D = 9$  equations of motion.

We now perform a  $D = 9$  to  $D = 8$  Scherk–Schwarz reduction with fluxes [20], making use of (combinations of) the global symmetries discussed above. We distinguish between the cases where  $\Lambda = 1$  ( $a = 0$ ) and where  $\Lambda \neq 1$  ( $a \neq 0$ ). Furthermore, we allow  $\Omega$  to be either the identity or an element of the three subgroups of  $SL(2, \mathbb{R})$ . Reduction to  $D = 8$  thus gives rise to eight different possibilities, one of which has to be split into two. These correspond to the nine  $D = 8$  maximal gauged supergravities with non-semisimple gauge groups, i.e. all Bianchi types except type VIII with gauge group  $SO(2, 1)$  and type IX with gauge group  $SO(3)$ . The result is given in table 2.

It can be seen that class A gauged supergravities are obtained by using only a subgroup of  $SL(2, \mathbb{R})$ , which is a reduction that can be performed on the  $D = 9$  ungauged action. Class B gauged supergravities, however, require the use of the extra  $SO(1, 1)$  symmetry which indeed can only be performed at the level of the field equations. The connection with  $D = 9$  clearly shows how it is possible to obtain the theories of class B from higher dimensions.

## 5. Domain wall solutions

In this section, we will focus on various solutions to the class A and class B supergravities in  $D = 8$  and also discuss the uplifting of these solutions to  $D = 11$ . For the class B supergravities we show that there are no domain wall solutions in  $D = 8$  that preserve any fraction of the supersymmetry. We do, however, find a cosmologically interesting (non-supersymmetric and time-dependent) spacelike domain wall solution, i.e. a domain wall solution where the single transverse direction is time.

In [10], we obtained the most general half supersymmetric domain wall solutions of the class A supergravities:

$$\begin{aligned}
 ds^2 &= H^{\frac{1}{12}} dx_7^2 - H^{-\frac{5}{12}} dy^2, \\
 e^\varphi &= H^{\frac{1}{4}}, \quad e^\sigma = H^{-\frac{1}{2\sqrt{3}}} h_1^{\frac{\sqrt{3}}{2}}, \quad e^\phi = H^{-\frac{1}{2}} h_1^{\frac{1}{2}} h_2, \\
 \chi_1 &= \chi_2 = \chi_3 = 0,
 \end{aligned} \tag{5.1}$$

<sup>8</sup> In the notation of [41], this corresponds to the combination  $\alpha - \frac{3}{4}\delta$ .

where the dependence on the transverse coordinate  $y$  is governed by

$$H(y) = h_1 h_2 h_3, \quad h_1 \equiv q_1 y + c_1, \quad h_2 \equiv q_2 y + c_2, \quad h_3 \equiv q_3 y + c_3. \quad (5.2)$$

Here  $c_m$  are arbitrary constants whose values will affect the range of  $y$ , due to the obvious requirement  $h_m > 0$ . The Killing spinor satisfies the condition

$$(1 + \Gamma_{y123})\epsilon = 0, \quad (5.3)$$

where the indices 1, 2, 3 refer to the internal group manifold directions. Note that the dependence on the transverse coordinate  $y$  is expressed in terms of three functions  $h_m$  which are harmonic on  $\mathbb{R}$ . We define  $n$  to be the number of linearly independent harmonics  $h_m$  with  $q_m \neq 0$ . The maximal value of  $n$  in a specific class is then given by the number of non-zero  $q_m$  of the corresponding structure constants. We call the solution an  $n$ -tuple domain wall<sup>9</sup> with  $n \leq 3$ . In this terminology,  $n = 3$  gives a triple,  $n = 2$  a double and  $n = 1$  a single domain wall, while  $n = 0$  is a flat spacetime [10].

Note that the solution (5.1) is given in an  $SL(3, \mathbb{R})$  frame where the three-dimensional gauge freedom has been fixed. The solution for all gauge choices is given in [10]. In addition to the gauge group, one can use the larger automorphism group (of which the gauge group with constant parameters is a subgroup) to set  $c_m = 1$  if  $q_m = 0$ . Furthermore, one parameter can be set to zero by shifting the transverse coordinate  $y$ . Thus, the number of parameters of the solution is  $n - 1$  (for  $n \geq 1$ ).

Upon uplifting to  $D = 11$ , using relation (3.2), we find that the  $n$ -tuple domain wall solutions become purely gravitational solutions with a metric of the form  $ds^2 = dx_7^2 - ds_4^2$ , where

$$ds_4^2 = H^{-\frac{1}{2}} dy^2 + H^{\frac{1}{2}} \left( \frac{\sigma_1^2}{h_1} + \frac{\sigma_2^2}{h_2} + \frac{\sigma_3^2}{h_3} \right). \quad (5.4)$$

Here  $\sigma^m$  are the Maurer–Cartan 1-forms defined in (3.7) and (3.9). The uplifted solutions are all 1/2 BPS except for the cases when  $h_1 = h_2 = h_3$  (only possible for Bianchi I and IX), which uplift to flat spacetime and thus become fully supersymmetric upon uplifting. Note that the solution (5.4) is an extension to different Bianchi types of the generalized Eguchi–Hanson solution constructed in [24].

We would like to see whether there are also supersymmetric domain wall solutions to the class B supergravities. It turns out that for this case there are no domain wall solutions preserving any fraction of supersymmetry. This can be seen as follows. The structure of the BPS equations requires the projector for the Killing spinor of a 1/2 BPS domain wall solution to be the same as above. The presence of the extra term in  $\delta\psi_\mu$  (see (3.16)), depending on the trace of the structure constants, implies that there are no domain wall solutions with this type of Killing spinor, since the structure of  $\Gamma$ -matrices of this term cannot be combined with other terms. To get a solution, one is forced to put  $f_{ij}{}^j = 0$ , thus leading back to the class A case. This also follows from  $\delta\lambda_i$ , since the resulting equation is symmetric in two indices, except for a single antisymmetric term, containing  $f_{ij}{}^j$ . Next, we search for domain wall solutions preserving an arbitrary fraction of the supersymmetry. From the structure of the BPS equations, it is seen that only one additional kind of projector is allowed, namely

$$(1 + \Gamma_{\alpha 123})\epsilon = 0, \quad (5.5)$$

<sup>9</sup> Compare this to, e.g., the D8-brane which is expressed in terms of one harmonic function,  $h = 1 + my$ , where the mass parameter  $m$  is piecewise constant. The domain walls are located at the points in  $y$  where  $m$  is discontinuous. In the same way, our  $n$  constituent domain walls will be located where the corresponding  $q_m$  change values. In [42], the double domain wall of [39] is given in a form similar to (5.1).

**Table 3.** The numbers of isometries of the three-dimensional group manifold for the different  $n$ -tuple domain wall solutions. For a given type one finds isometry enhancement by decreasing  $n$ , i.e. upon identifying two harmonic functions  $h_m$ .

Bianchi	$(q_1, q_2, q_3)$	$n = 0$	$n = 1$	$n = 2$	$n = 3$
I	(0, 0, 0)	6	–	–	–
II	(0, 0, 1)	–	4	–	–
VI <sub>0</sub>	(0, –1, 1)	–	–	3	–
VII <sub>0</sub>	(0, 1, 1)	–	6	3	–
VIII	(1, –1, 1)	–	–	4	3
IX	(1, 1, 1)	–	6	4	3

where  $\alpha \neq y$  and spacelike. However, this again leads to  $f_{ij}{}^j = 0$ . We conclude that there are no domain wall solutions preserving any fraction of supersymmetry for the class B supergravities.

As we have shown in section 3, the internal three-dimensional manifolds are by definition invariant under the three-dimensional group of isometries given in (3.14) and (3.15). This holds for arbitrary values of the scalars in (3.11). However, there can be more isometries, which rotate two of the Maurer–Cartan 1-forms  $\sigma^m$  and  $\sigma^p$  into each other. This is an isometry of the metric in two cases.

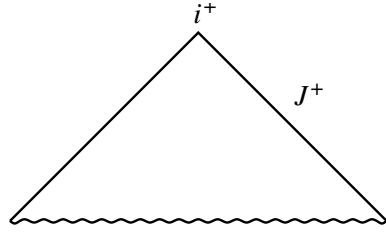
- $q_m = q_p = 0$ . In this case one can use the automorphism group to set  $c_m = c_p = 1$ . Equation (5.4) shows that a rotation between  $\sigma^m$  and  $\sigma^p$  is an isometry for all solutions of this class.
- $q_m = q_p \neq 0$ . In this case one must set  $c_m = c_p$  by hand, after which a rotation between  $\sigma^m$  and  $\sigma^p$  is an isometry. Thus, this only holds for a truncation of the solutions of this class and since  $h_m = h_p$  corresponds to decreasing  $n$  by one.

This leads to the different possibilities summarized in table 3. Note that these exhaust all possible numbers of isometries on three-dimensional class A group manifolds as given in table 1. The extra fourth isometry was constructed by Bianchi [25] for the types II, VIII and IX. He claimed that type VII<sub>0</sub> did not allow for isometry enhancement but the existence of three extra Killing vectors<sup>10</sup> was later shown in [14, 36, 37]. These three extra isometries appear upon identifying the two  $y$ -dependent harmonics. Note that the extra isometries may not be isometries of the full manifold in which the group submanifold is embedded. Indeed, this is what happens for type VII<sub>0</sub> where two of the extra isometries are  $y$  dependent and therefore do not leave the full metric invariant [14, 36, 37]. The extra Killing vectors of the group manifold for the uplifted domain wall solutions (5.1) are explicitly given in appendix B for completeness.

As we have mentioned above, two of the class A solutions uplift to flat spacetime in  $D = 11$ : the Bianchi type IX solutions with  $n = 1$  and all Bianchi type I solutions (having  $n = 0$ ). In view of the above discussion, we can now understand why this happens. One can check that the only way to embed three-dimensional submanifolds of zero (for type I) or constant positive (for type IX) curvature in four Euclidean Ricci-flat dimensions is to embed them in four-dimensional flat space. Indeed, this is exactly what we find: the two solutions both have a maximally symmetric group manifold with six isometries and hence constant curvature and uplift to flat  $D = 11$  spacetime.

The type VII<sub>0</sub> group manifold can also have six isometries and zero curvature. For the domain wall solutions given above, this cannot be embedded in four-dimensional flat space

<sup>10</sup> We thank Sigbjørn Hervik for a valuable discussion on this point.



**Figure 1.** The Penrose diagram for the solution (5.6), and for the Einstein–de Sitter universe, is given by the upper half of the diamond representing Minkowski spacetime and has a singularity at  $t = 0$ .

due to the  $y$ -dependence of two of its isometries. Note, however, that there is another type  $\text{VII}_0$  solution with flat geometry and vanishing scalars that coincides with the type I solution (5.1) given above<sup>11</sup>. The corresponding group manifold can be embedded in four-dimensional flat space and indeed this solution uplifts to 11-dimensional Minkowski just as the type I solution. However, unlike its type I counterpart, the eight-dimensional type  $\text{VII}_0$  solution with flat geometry and vanishing scalars breaks all supersymmetry.

When we include the class B supergravities, we deduce from table 1 that there are two more cases with maximally symmetric group manifolds, which have constant negative curvature, namely type V and type  $\text{VII}_a$ . The group manifold can only be embedded in a four-dimensional Ricci-flat manifold if the embedding space is flat and Lorentzian. Thus one can expect solutions of Bianchi type V and type  $\text{VII}_a$  that have six isometries and uplift to flat spacetime in  $D = 11$ . It is interesting to find out what these extra solutions look like. By solving the field equations in  $D = 11$  using the Bianchi type V or type  $\text{VII}_a$  ansatz with constant coset scalars  $\mathcal{M}_{mn}$ , we find the following (non-supersymmetric and time-dependent) solution in  $D = 8$ :

$$ds^2 = dt^2 - t^{2/3} dx_7^2, \quad e^\varphi = \frac{9}{4} t^2, \quad (5.6)$$

where all scalars except  $\varphi$  have been put to zero using the automorphism groups of the Bianchi type V and type  $\text{VII}_a$  algebras. One can view this as a spacelike domain wall, i.e. a domain wall where the single transverse direction is time. There are no (non-supersymmetric) static domain wall solutions for constant coset scalars  $\mathcal{M}_{mn}$ .

The solution (5.6) describes an expanding universe with the same qualitative features as those of the Einstein–de Sitter universe<sup>12</sup>. In the present case the stress–energy tensor is generated by the scalar field  $\varphi$ . The  $D = 8$  Ricci scalar is given by  $R = \frac{14}{9} t^{-2}$ . Note that the metric (5.6) can be rewritten as being conformal to Minkowski spacetime, by the coordinate change  $\tau \sim t^{2/3}$ . The Penrose diagram for the solution (5.6) is therefore given by the upper half of the diamond that represents Minkowski spacetime with a singularity at  $t = 0$  (see figure 1).

Upon uplifting to  $D = 11$  (and rescaling  $t$ ), we get the following solution:

$$d\hat{s}^2 = dt^2 - dx_7^2 - \frac{9}{4} t^2 \sigma^m \sigma^m, \quad (5.7)$$

where summation over  $m$  is understood. The Maurer–Cartan 1-forms  $\sigma^m$  are defined in (3.7) and (3.9). The metric (5.7) is a 7D flat Euclidean metric times a 4D metric with Lorentz signature which turns out to be a particular parametrization of flat spacetime. Thus, we

<sup>11</sup> This solution coincides, after an  $SO(2)$  rotation of  $90^\circ$ , with the Kaluza–Klein reduction of the  $\text{Mink}_9$  solution [43, 41] of the  $SO(2)$  gauged supergravity in  $D = 9$ .

<sup>12</sup> The Einstein–de Sitter universe is a flat ( $k = 0$ ) matter-dominated ( $p = 0$ ) Robertson–Walker spacetime with zero cosmological constant ( $\Lambda = 0$ ).

indeed find that the eight-dimensional solution uplifts to the maximally (super-)symmetric flat spacetime in  $D = 11$  as expected. If we instead reduce on the seven-dimensional group manifold obtained by taking a Bianchi type V or  $VII_a$  manifold times  $T^4$ , we get the Einstein–de Sitter universe as a solution<sup>13</sup> in  $D = 4$ . This solution also uplifts to flat spacetime in  $D = 11$ . Note that the equations of motion in both  $D = 8$  and  $D = 4$  cannot be obtained from an action. The nice feature of obtaining Einstein–de Sitter universes in this way is that they have a very simple and natural higher-dimensional origin, namely the only maximally (super-)symmetric vacuum solution in  $D = 11$ , i.e. flat spacetime.

## 6. Conclusions

In this work we have constructed 11 maximal  $D = 8$  gauged supergravities in terms of the Bianchi classification of three-dimensional Lie groups, which distinguishes between class A and class B. We find that this distinction carries over to a number of features of the eight-dimensional theories. Class A theories can be formulated in terms of an action, whereas the theories of class B have equations of motion that cannot be integrated to an action. Moreover, only the supergravities of class A admit 1/2 BPS domain wall solutions. These solutions provide realizations of isometry enhancement in the group manifold after identification of the harmonics. The three solutions that have a maximum number of isometries uplift to  $D = 11$  flat spacetime.

We find that there are no domain wall solutions for the class B theories that preserve any supersymmetry. However, we have found a (non-supersymmetric and time-dependent) spacelike domain wall solution to two of the class B theories. The solution describes an expanding universe with the same qualitative features as those of the Einstein–de Sitter universe. By instead reducing over a seven-dimensional group manifold we obtain the Einstein–de Sitter universe as a solution in  $D = 4$ . Both solutions uplift to the only maximally (super-)symmetric vacuum solution in  $D = 11$ , i.e. flat spacetime, which provides a nice higher-dimensional origin of Einstein–de Sitter universes.

The Einstein–de Sitter solution has an interesting cosmological interpretation. It has recently been argued that Einstein–de Sitter models are acceptable models of the universe [26], and, e.g., fit the CMB data equally well if not better than the best concordance model. This, however, assumes that there must be some other explanation of the observed Hubble diagram of distant type Ia supernovae [44] than a positive cosmological constant. It would be interesting to investigate further the occurrence of Einstein–de Sitter universes in compactifications of M-theory.

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<sup>13</sup> We have to go to a particular frame in order to get exactly the Einstein–de Sitter solution in  $D = 4$ . In, e.g., the Einstein frame we instead find a conformally related solution. We can in this way also obtain Einstein–de Sitter-like solutions for all  $D \leq 8$ .



## Appendix A. Supersymmetry rules

In this appendix we give the full supersymmetry rules (up to higher-order fermions) of all  $D = 8$  class A and class B supergravities. Considering the ansatz (3.3), the dimensional reduction of the 11-dimensional field strength  $\hat{G}$  leads to the eight-dimensional field strengths

$$\begin{aligned} G_{\mu\nu\rho\lambda} &= 4\partial_{[\mu} C_{\nu\rho\lambda]} + 6F^m_{[\mu\nu} B_{\rho\lambda]m}, \\ G_{\mu\nu\rho m} &= 3\mathcal{D}_{[\mu} B_{\nu\rho]m} + 3F^n_{[\mu\nu} V_{\rho]mn}, \\ G_{\mu\nu mn} &= 2\mathcal{D}_{[\mu} V_{\nu]mn} - f_{mn}{}^p B_{\mu\nu p} + \ell\epsilon_{mnp} F^p_{\mu\nu}, \\ G_{\mu mnp} &= \epsilon_{mnp} \partial_\mu \ell + 3(V_{\mu r[m} + \ell A^q_{\mu} \epsilon_{qr[m}]) f_{np]}{}^r, \end{aligned} \quad (\text{A.1})$$

where the field strength of the gauge field is given by

$$F^m_{\mu\nu} = 2\partial_{[\mu} A^m_{\nu]} - f_{np}{}^m A^p_\mu A^n_\nu. \quad (\text{A.2})$$

The curvatures (A.1) are invariant under the gauge transformations that arise upon reduction of the  $D = 11$  law  $\delta\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3\partial_{[\hat{\mu}} \hat{\Lambda}_{\hat{\nu}\hat{\rho}]}$ . Using the ansatz

$$\begin{aligned} \hat{\Lambda}_{\mu\nu} &= \Lambda_{\mu\nu} - 2A^m_{[\mu} \Lambda_{\nu]m} + A^m_\mu A^n_\nu \Lambda_{mn}, \\ \hat{\Lambda}_{\mu m} &= U^q{}_m (\Lambda_{\mu q} - A^n_\mu \Lambda_{qn}), \\ \hat{\Lambda}_{mn} &= U^p{}_m U^q{}_n \Lambda_{pq}, \end{aligned} \quad (\text{A.3})$$

the gauge transformations in  $D = 8$  are

$$\begin{aligned} \delta C_{\mu\nu\rho} &= 3\partial_{[\mu} \Lambda_{\nu\rho]} - 3F^m_{[\mu\nu} \Lambda_{\rho]m}, \\ \delta B_{\mu\nu m} &= 2\mathcal{D}_{[\mu} \Lambda_{\nu]m} - \Lambda_{mn} F^m_{\mu\nu}, \\ \delta V_{\mu mn} &= \mathcal{D}_\mu \Lambda_{mn} + f_{mn}{}^p \Lambda_{\mu p}, \\ \delta \ell &= \frac{1}{2} \epsilon^{mnp} f_{mn}{}^q \Lambda_{qp}. \end{aligned} \quad (\text{A.4})$$

The supersymmetry transformation rules in eight dimensions are

$$\begin{aligned} \delta e_\mu{}^a &= -\frac{i}{2} \bar{\epsilon} \Gamma^a \psi_\mu \\ \delta \psi_\mu &= 2\partial_\mu \epsilon - \frac{1}{2} \phi_\mu \epsilon + \frac{1}{2} L_{[i}{}^m \mathcal{D}_\mu L_{m]j} \Gamma^{ij} \epsilon + \frac{1}{24} e^{-\varphi/2} f_{ijk} \Gamma^{ijk} \Gamma_\mu \epsilon - \frac{1}{6} e^{-\varphi/2} f_{ij}{}^j \Gamma_\mu \Gamma^i \epsilon \\ &\quad + \frac{1}{24} e^{\varphi/2} \Gamma^i L_i{}^m (\Gamma_\mu{}^{\nu\rho} - 10\delta_\mu{}^\nu \Gamma^\rho) F_{m\nu\rho} \epsilon - \frac{i}{12} e^{-\varphi} \Gamma^{ijk} L_i{}^m L_j{}^n L_k{}^p G_{\mu mnp} \epsilon \\ &\quad + \frac{i}{96} e^{\varphi/2} (\Gamma_\mu{}^{\nu\rho\delta\epsilon} - 4\delta_\mu{}^\nu \Gamma^{\rho\delta\epsilon}) G_{\nu\rho\delta\epsilon} \epsilon + \frac{i}{36} \Gamma^i L_i{}^m (\Gamma_\mu{}^{\nu\rho\delta} - 6\delta_\mu{}^\nu \Gamma^{\rho\delta}) G_{\nu\rho\delta m} \epsilon \\ &\quad + \frac{i}{48} e^{-\varphi/2} \Gamma^i \Gamma^j L_i{}^m L_j{}^n (\Gamma_\mu{}^{\nu\rho} - 10\delta_\mu{}^\nu \Gamma^\rho) G_{\nu\rho mn} \epsilon, \\ \delta \lambda_i &= \frac{1}{2} L_i{}^m L^{jn} \mathcal{D} \mathcal{M}_{mn} \Gamma_j \epsilon - \frac{1}{3} \not{\partial} \phi \Gamma_i \epsilon - \frac{1}{4} e^{-\varphi/2} (2f_{ijk} - f_{jki}) \Gamma^{jk} \epsilon \\ &\quad + \frac{1}{4} e^{\varphi/2} L_i{}^m \mathcal{M}_{mn} F^n \epsilon + \frac{i}{144} e^{\varphi/2} \Gamma_i \mathcal{G} \epsilon + \frac{i}{36} (2\delta_i{}^j - \Gamma_i{}^j) L_j{}^m \mathcal{G}_m \epsilon \\ &\quad + \frac{i}{24} e^{-\varphi/2} \Gamma^j L_j{}^m L_k{}^n (3\delta_i{}^k - \Gamma_i{}^k) \mathcal{G}_{mn} \epsilon + \frac{i}{6} e^{-\varphi} \Gamma^{jk} L_i{}^m L_j{}^n L_k{}^p \mathcal{G}_{mnp} \epsilon, \\ \delta A^m_\mu &= -\frac{i}{2} e^{-\varphi/2} L_i{}^m \bar{\epsilon} \left( \Gamma^i \psi_\mu - \Gamma_\mu \left( \eta^{ij} - \frac{1}{6} \Gamma^i \Gamma^j \right) \lambda_j \right), \end{aligned}$$

$$\begin{aligned}
\delta V_{\mu mn} &= \varepsilon_{mnp} \left[ -\frac{i}{2} e^{\varphi/2} L_i{}^p \bar{\epsilon} \left( \Gamma^i \psi_\mu + \Gamma_\mu \left( \eta^{ij} - \frac{5}{6} \Gamma^i \Gamma^j \right) \lambda_j \right) - \ell \delta A^p{}_\mu \right], \\
\delta B_{\mu\nu m} &= L_m{}^i \bar{\epsilon} \left( \Gamma_{i[\mu} \psi_{\nu]} + \frac{1}{6} \Gamma_{\mu\nu} (3\delta_i{}^j - \Gamma_i \Gamma^j) \lambda_j \right) - 2\delta A^n{}_{[\mu} V_{\nu]mn}, \\
\delta C_{\mu\nu\rho} &= \frac{3}{2} e^{-\varphi/2} \bar{\epsilon} \Gamma_{[\mu\nu} (\psi_{\rho]} - \frac{1}{6} \Gamma_{\rho]} \Gamma^i \lambda_i) - 3\delta A^m{}_{[\mu} B_{\nu\rho]m}, \\
L_i{}^n \delta L_{nj} &= \frac{i}{4} e^{\varphi/2} \bar{\epsilon} \left( \Gamma_i \delta_j{}^k + \Gamma_j \delta_i{}^k - \frac{2}{3} \eta_{ij} \Gamma^k \right) \lambda_k, \\
\delta\varphi &= -\frac{i}{2} \bar{\epsilon} \Gamma^i \lambda_i, \\
\delta\ell &= -\frac{i}{2} e^\varphi \bar{\epsilon} \Gamma^i \lambda_i.
\end{aligned} \tag{A.5}$$

## Appendix B. Killing vectors

In this appendix we give the Killing vectors associated with the isometry enhancement taking place for some of the domain wall solutions, as discussed in section 5.

### B.1. Class A

For the class A solutions we denote the extra Killing vectors  $X_4$ ,  $X_5$  and  $X_6$ , corresponding to rotations between  $\sigma^1$  and  $\sigma^2$ ,  $\sigma^1$  and  $\sigma^3$  and  $\sigma^2$  and  $\sigma^3$ , respectively.

- Type I with  $Q = \frac{1}{2} \text{diag}(0, 0, 0)$ :

$$X_4 = -z^2 \frac{\partial}{\partial z^1} + z^1 \frac{\partial}{\partial z^2}, \quad X_5 = -z^3 \frac{\partial}{\partial z^1} + z^1 \frac{\partial}{\partial z^3}, \quad X_6 = -z^3 \frac{\partial}{\partial z^2} + z^2 \frac{\partial}{\partial z^3}. \tag{B.1}$$

- Type II with  $Q = \frac{1}{2} \text{diag}(0, 0, 1)$ :

$$X_4 = -z^2 \frac{\partial}{\partial z^1} + z^1 \frac{\partial}{\partial z^2} + \frac{1}{4} ((z^1)^2 - (z^2)^2) \frac{\partial}{\partial z^3}. \tag{B.2}$$

- Type VII<sub>0</sub> with  $Q = \frac{1}{2} \text{diag}(0, 1, 1)$  with  $h(y) = h_2 = h_3$ :

$$\begin{aligned}
X_4 &= -h^{-1/2} z^2 \frac{\partial}{\partial z^1} + h^{1/2} z^1 \frac{\partial}{\partial z^2}, \\
X_5 &= -h^{-1/2} z^3 \frac{\partial}{\partial z^1} + h^{1/2} z^1 \frac{\partial}{\partial z^3}, \\
X_6 &= -z^3 \frac{\partial}{\partial z^2} + z^2 \frac{\partial}{\partial z^3}.
\end{aligned} \tag{B.3}$$

- Type VIII with  $Q = \frac{1}{2} \text{diag}(1, -1, 1)$ :

$$X_5 = \frac{s_{3,2,1} s_{1,3,2}}{c_{1,3,2}} \frac{\partial}{\partial z^1} + c_{3,2,1} \frac{\partial}{\partial z^2} + \frac{s_{3,2,1}}{c_{1,3,2}} \frac{\partial}{\partial z^3}. \tag{B.4}$$

- Type IX with  $Q = \frac{1}{2}\text{diag}(1, 1, 1)$ :

$$\begin{aligned} X_4 &= -\frac{c_{3,2,1}s_{1,3,2}}{c_{1,3,2}}\frac{\partial}{\partial z^1} + s_{2,3,1}\frac{\partial}{\partial z^2} - \frac{c_{3,2,1}}{c_{1,3,2}}\frac{\partial}{\partial z^3}, \\ X_5 &= \frac{s_{3,2,1}s_{1,3,2}}{c_{1,3,2}}\frac{\partial}{\partial z^1} + c_{3,2,1}\frac{\partial}{\partial z^2} + \frac{s_{3,2,1}}{c_{1,3,2}}\frac{\partial}{\partial z^3}, \\ X_6 &= -\frac{\partial}{\partial z^1}. \end{aligned} \quad (\text{B.5})$$

## B.2. Class B

For class B there are scalings associated with the non-zero parameter  $a$ , and therefore the extra Killing vectors do not correspond to just rotations among the Maurer–Cartan 1-forms in this case.

- Type V with  $Q = \frac{1}{2}\text{diag}(0, 0, 0)$  and  $a = 1$ :

$$\begin{aligned} X_4 &= -z^2\frac{\partial}{\partial z^1} + \frac{1}{2}((z^2)^2 - (z^3)^2 - e^{-2z^1})\frac{\partial}{\partial z^2} + z^2z^3\frac{\partial}{\partial z^3}, \\ X_5 &= -z^3\frac{\partial}{\partial z^1} + z^2z^3\frac{\partial}{\partial z^2} + \frac{1}{2}((z^3)^2 - (z^2)^2 - e^{-2z^1})\frac{\partial}{\partial z^3}, \\ X_6 &= -z^3\frac{\partial}{\partial z^2} + z^2\frac{\partial}{\partial z^3}. \end{aligned} \quad (\text{B.6})$$

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